605

VOLUME 81 SEPARATE No. 605

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

JANUARY, 1955

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DIVISION

(Discussion open until May 1, 1955)

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

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This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

IMPACT ON PINJOINTED TRUSSES

Bruno A. Boley[1] and Chi-Chang Chao[2]

SUMMARY

A method for the analysis of pin-jointed trusses of arbitrary configuration subjected to impact at some of their joints, is presented in this paper. Expressions are first derived describing the behavior of the various types of joints which may be present in a truss; with the aid of these, a numerical calculation procedure is developed. This procedure makes use of "distribution factors," dependent on the joint geometry only, to describe the relation between the waves arising in the bars meeting at an unloaded joint; the analogous relations at a loaded joint is determined with the aid of a finite-differences technique. The method suggested can be used whether impact is caused by a known pressure pulse or by collision with a solid object. The criterion for establishing the time at which impact ends is discussed in detail for the latter of these cases. An illustrative example is presented, for which stresses, deflections and reactions are calculated, the proposed numerical procedure being followed in detail. The basic equations governing the behavior of the various joints are derived for three-dimensional, as well as plane, trusses; the proposed method of analysis is equally applicable to both.

INTRODUCTION

The behavior of pin-jointed trusses under time-dependent loadings is discussed in this paper by means of a dynamic analysis, which considers in detail the elastic waves set up in the various bars of the truss. Such an analysis is necessary if the loads are rapidly applied, either by means of a sudden pressure wave or through collision with a fast moving solid object. Of course, if the loads are very gradually applied, the customary static analysis (or an analysis taking into account the excitation of the first few normal modes of vibration) may be adequate; as a matter of fact, this question is being studied in more detail in another part of the present investigation. The types of loadings mentioned above, however, are becoming increasingly important in a number of engineering applications and therefore call for a more accurate method of calculation. The purpose of this investigation is therefore the development of a procedure suitable for practical use, and at the same time general enough to include structures of arbitrary configuration.

The paper is divided into six sections, as follows:

1. General Discussion. A general discussion of the problem and an outline of the method of solution are given. The next section, namely,

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2. Equations for Various Types of Joints presents the basic formulas (and their derivation) for the solution procedure previously described for the case of plane trusses. The following five types of joints are taken up in detail:

Type (a). Loaded joint; all waves travelling away from the joint.
Type (b). Interior unloaded joint, after the arrival of a wave in one of the bars.
Type (c). Loaded joint, after the arrival of a wave in one of the bars.
Type (d). Joint with roller support.
Type (e). Joint with fixed hinge support.

3. The Criterion for the End of Impact is discussed in this section, and is needed for the cases in which the loading is produced by physical impact with a solid object. Use of this criterion, and of the entire procedure of section 1 is illustrated in section 4, i.e.

4. Numerical Solution and Illustrative Example, with the aid of a plane truss of arbitrary configuration.

5. Some Additional Remarks are then presented, referring to loading conditions other than those of section 3, checks on the calculations, etc.

6. Equations for Space Trusses are given in this section.

The present investigation is part of a project sponsored by the Office of Naval Research, U. S. Navy.[3] The contributions of Mr. Robert E. Heninger to the present work are gratefully acknowledged.

List of Symbols

a	speed of wave propagation, i.e. $a = (E/\rho)^{1/2}$
A_i	cross-sectional area of i-th bar
C_1, C_2	constants
D	distribution factor, see Tables I and II and eqs. (33).
E	Young's modulus
f_i	wave in bar i
$f_{ijkl\ldots yz}$	fraction of the wave starting in bar i, resulting from the successive distributions at the joints where bars i and j meet, bars j and k meet,....., and finally at the joint where bars y and z meet.
f_{Ii}, f_{Oi}	total incoming and outgoing waves, respectively, in bar i at the loaded joint (type c).
f_{pBi}	wave caused in bar i by the total wave entering joint B through bar p, when joint B is of type (c).
F	force
$i, j, k\ldots.$	subscripts indicating bars
L	characteristic length; taken as unity in the numerical example.
L_i	length of i-th bar
n_i	$= M/(\rho A_i L_i)$

3. The results described herein were first presented in Technical Reports No. 1, "The Analysis of Plane Trusses under Impact," by Bruno A. Boley and Chi-Chang Chao (1953) and No. 3 "Equations for Three-Dimensional Trusses Under Impact," by Chi-Chang Chao and Bruno A. Boley (1954) issued under contract Nonr-266(20), at Columbia University.

M	mass of striking body
n	number of bars meeting at a joint
p	subscript indicating the bar travelled by incoming wave
P_1, P_2, P_3	constants defined in eqs. (15a), (15b) and (32a), respectively
r_1, r_2	$= (1/2\left[-P \pm \sqrt{P_1^2 - 4P_2}\right]$
R	reaction
t	time
u_i	axial displacement in the i-th bar
V	velocity
x_i	distance along i-th bar
δ	displacement
θ_{ij}, θ_{iV}, θ_{iF}	angle between bar i and bar j, velocity V and force F_1, respectively
ρ	mass density
σ_i	normal axial stress in bar i
τ	$= at/L$
$\Delta\tau$	interval of τ used with finite-differences solution; = 0.2 for the numerical example

1. General Discussion

A general description of the problem and of the method of solution is presented in this section. This method consists essentially of a procedure for following the waves set up, when impact occurs, in the various bars of the truss and of determining how these waves are distributed among the bars meeting at any one joint. For a truss with arbitrary configuration or with a large number of joints and bars this procedure can become very cumbersome unless it is carried out in a systematic manner. It is therefore desirable to consider first separately the various sources of difficulties, and to determine how each may best be treated.

The first requirement of the calculation procedure is a method of considering all the waves at any one time at whatever joint is considered. It was found that this is best accomplished by solving first for all the waves affecting the loaded joint, and subsequently extending the solution to the other portions of the truss. The calculation of the waves at the loaded joint is in turn broken up in three phases:

(a) only waves emanating from the loaded joint are present, before they reach the neighboring joints,
(b) distribution of these waves among the bars at the neighboring joints,
(c) arrival and distribution of these new waves at the loaded joint.

The actual steps of a procedure in which the above phases are followed is described in detail in steps 2-6 of section 4.

In the second place it is necessary to determine how the waves are distributed at any one joint. The distribution process will obey two rules: (1) the vector sum of the bar forces must vanish (for an unloaded joint) or balance the inertia force of the striking mass (at a loaded joint); (2) the velocity of each bar must be compatible with the condition that all bars remain attached at the joint. These two conditions, one on the stresses and one on the velocity, could be simplified if a simple relation existed between the stress and the velocity of any one bar. In phase (a) of the previous paragraph the stress and velocity of the end points of each bar will be found to be proportional, while in phases (b) and (c) this is true only for a portion of the distributed

waves; the remaining portion is proportional to the incoming wave and is therefore known. These matters are expressed mathematically in eq. (4) for phase (a) and in eqs. (8b) and (8c) for phases (b) and (c); these are employed in the outline contained in section 4.

In the third place, it will be found (see eqs. (9)) that the distribution of waves at an unloaded joint (phase (b)) is readily performed, while that at a loaded joint (phase (c)) requires the use of a finite-differences technique. For this reason these phases are kept separate in the final procedure.

The general ideas indicated above form the basis of the proposed solution; the remainder of this section will elaborate on them, introduce a suitable notation, and express the various phases of the problem in mathematical form. To fix ideas this will be done with reference to the particular truss of Fig. 1, but it will be clear that this in no way restricts the generality of the method. The discussion of this section is in fact valid for either two or three-dimensional trusses. The formulas for the distribution of waves at the various types of joints are given in section 2 for two-dimensional trusses, and in section 6 for three-dimensional ones.

Consider the truss of Fig. 1 to be struck at joint B, say, by a mass M with a velocity V in the direction shown. Waves immediately arise in all the bars meeting at that joint; the corresponding axial displacement u_i in the i-th bar satisfies the wave equation[4]

$$\frac{\partial^2 u_i}{\partial t^2} = a^2 \frac{\partial^2 u_i}{\partial x_i^2} \qquad (1)$$

where x_i is the axial distance along bar i, positive when measured from the joint in question. Bending deflections are neglected in this development.[5] At first only waves travelling away from the joint are present; (phase (a) above), hence

$$u_i = f_i(at - x_i) \qquad (2)$$

for any bar, where f_i is an arbitrary function. The stress

$$\sigma_i = \left(\frac{\partial u_i}{\partial x_i}\right) E \qquad (3)$$

and velocity $(\partial u_i/\partial t)$ in bar i at $x_i = 0$ (struck end, here joint B) are then connected by the relation

$$\frac{\partial u_i}{\partial x_i} = -\frac{1}{a}\frac{\partial u_i}{\partial t} = -f_i'(at) \qquad (4)$$

4. Love, A. E. H.: The Mathematical Theory of Elasticity. Fourth Edition, Dover Publications, New York, 1944.
5. This type of deflection is probably of little importance in most applications. It would of course become significant if the applied loads were such as to excite greatly the lower normal modes of vibration which involve considerable transverse deflections; in such problems a more thorough analysis would be required.

where primes indicate differentiation with respect to the argument of the function f_i

The mass M is acted upon by the stress of eq. (3), and behaves in accordance with Newton's law of motion, which, written in the direction of each of the bars, i

$$M \frac{\partial^2 u_i}{\partial t^2} = \sum_{j=1}^{n} A_j \sigma_j \cos\theta_{ij} \qquad i = 1, 2, 3, \cdots n \tag{5}$$

where all quantities are evaluated at $x_i = o$. A joint of the truss which obeys eqs (2) and (5) is a loaded joint with all waves travelling away from it and will be called of type (a). With the subsequent arrival of reflected waves this will no longer be true, and the joint will then be one of type (c).

Eqs. (5) assumes that the mass and the struck joint remain in contact; if contact were to cease, this equation would reduce to

$$\sum_{j=1}^{n} A_j \sigma_j \cos\theta_{ij} = 0 \qquad i = 1, 2, 3 \cdots n \tag{6}$$

A joint which obeys eq. (6) will be called of type (b).

Return now to the struck joint, and follow the waves proceeding from it along the various bars. The solution of eqs. (2) and (5) completely determines the wave in any bar (say the i-th) until a time $t = L_i/a$, that is when the wave has reached the far end of the bar in question. Consider for example bar 2 in the truss of Fig. 1; its wave will first affect joint C at $t = L_2/a$, by striking it with wave f_2 of eq. (2). If the origin is now shifted to joint C, the magnitude of f_2 becomes

$$u_p = -f_p(at - L_p + x_p) \tag{7}$$

where p = 2 in the present example. In general, the subscript p will denote the bar along which a wave arrives. The wave f_p causes disturbances in all bars emanating from joint C, and travelling away from it (phase (b) above); hence, they all satisfy eq. (2). The governing equations for this joint are obviously (6); therefore, this joint is of type (b), and these new waves can be determined from those equations. Let the new wave in bar 4 be denoted by f_{24} (or, in general, the wave in bar i by f_{pi} then $u_4 = f_{24}$ (at - L_2 x_4), or in general

$$u_i = f_{pi}(at - L_p - x_i); \; f_{pi}(0) = 0, (i \neq p) \tag{8}$$

The corresponding equation f or bar p must, in addition, take into account the incoming wave of eq. (7) and is

$$u_p = -f_p(at - L_p + x_p) + f_{pp}(at - L_p - x_p) \tag{8a}$$

It follows from the above equations that, at $x_i = 0$,

$$\frac{\partial u_i}{\partial t} = \begin{cases} a f'_{pi} & p \neq i \\ a(f'_{pp} - f'_p) & p = i \end{cases} \tag{8b}$$

and

$$\frac{\sigma}{E} = \frac{\partial u_i}{\partial x_i} = \begin{cases} -\frac{1}{a}\frac{\partial u_i}{\partial t} = -f'_{pi} & p \neq i \\ -f'_p - f'_{pp} = \frac{1}{a}\frac{\partial u_i}{\partial t} + 2f'_p & p = i \end{cases} \tag{8C}$$

Notice that in all bars but the p-th eqs. (4) still hold; for the p-th bar only a portion of the stress is proportional to the velocity, while the remainder is proportional to the known incoming wave f'_p.

Solution of eqs. (6) will show that the simple relations

$$f'_{pi} = D_{pi} f'_p \tag{9}$$

holds for all bars, where D_{pi} is a "distribution factor" dependent only on the geometries of the joint in question and its bars; it is listed in Table I. Note that, in general, the relation

$$D_{ip} = D_{pi} \tag{9a}$$

holds, and that D_{pp} will in general be different when calculated at different ends of the same bar. Thus (see Fig. 1) $D_{44C} \neq D_{44D}$, for example.

Let us now continue to follow the new waves just started at joint C. Of these, f_{22} will reach joint B at $t = 2L_2/a$; f_{24} will reach joint D at $t = (L_2 + L_4)/a$ and so forth. The wave f_{24} will distribute at joint D (which is also of type (b)), according to the distribution factor appropriate to that joint; it will thus give rise to the following new waves (travelling away from joint D):

$$\begin{aligned} f'_{243}(at - x_3) &= D_{24} D_{43} f'_2(at - L_2 - L_4 - x_3) \\ f'_{244}(at - x_4) &= D_{24} D_{44D} f'_2(at - L_2 - L_4 - x_4) \\ f'_{247}(at - x_7) &= D_{24} D_{47} f'_2(at - L_2 - L_4 - x_7) \end{aligned} \tag{10}$$

where x_3, x_4, and x_7 are now measured from joint D. Thus, when at $= (L_2 + L_4 + L_3)$ wave f_{243} reaches the loaded joint B.

Wave f_{342} will of course reach joint B at the same time as f_{243} (note $f_{342} \neq f_{243}$); but before that happens it has been seen that f_{22} has already arrived there, and similarly so will have f_{11} and f_{33}. With the exception of f_{11}, all these waves encounter only joints of type (b) and therefore behave as

described above. Joint A is a fixed hinged support, and therefore cannot displace; such a joint will be called of type (c) and can be treated in a manner analogous to that of type (b) with the use of the appropriate distribution factors listed in table I. Incidentally, the same is true of joints of type (d), that is joints with a roller support; Table I gives the distribution factors for this case as well.

There remains now to consider the behavior of the loaded joint (B in this case) after the arrival of waves along its component bars (phase (c) above). This was defined as a joint of type (c) and is a combination of types (a) and (b) in that it follows eqs. (5) and (8). The solution of these equations cannot be put in terms distribution factors similar to those introduced above; the governing relations are derived in the next section. It will be seen that it is most advantageous to employ a finite-differences technique in their solution.

The remarks given above provide the basis for the proposed method of analysis. The derivation of the basic equations is given in the next section, while the details of the procedure to be followed in the solution of an actual problem are presented, together with an illustrative example, in section 4.

2. Equations for Various Types of Joints

The basic equations for each type of joint are derived below for the case of a plane truss.

Type (a). In the case each joint has two degrees of freedom; hence, there can be only two independent equations in the set (5). Take, for example, the i-th and j-th equations ($i \neq j$) as the independent ones; then any other (say the k-th) may be obtained from them by multiplying the i-th equation by ($\sin\theta_{jk}/\sin\theta_{ij}$), the j-th by ($\sin\theta_{ki}/\sin\theta_{ij}$), adding the results and simplifying the expression thus obtained with the aid of the equations

$$\theta_{ii} = 0 \; ; \quad \theta_{ij} + \theta_{ji} = 2\pi \; ; \quad \theta_{ij} + \theta_{jk} = \theta_{ik} \; ; \; etc. \tag{11}$$

and the compatibility relations

$$u_i \quad \sin\theta_{jk} + u_j \sin\theta_{ki} + u_k \sin\theta_{ij} = 0 \; ; \quad (x_i = x_j = x_k = 0) \tag{12}$$

In two special cases (k = i and k = j) eq. (12) is trivial; in the remaining (n-2) cases it may be easily derived by equating the two expressions for the distance PQ given in Fig. 2. Eq. (12) holds also if u_i is replaced by ($\partial u_i/\partial t$); then use of eq. (4) shows that one may also write

$$f_i' \sin\theta_{jk} + f_j' \sin\theta_{ki} + f_k' \sin\theta_{ij} = 0 \tag{12a}$$

With (12a) and (4) all displacements but two may be eliminated from the two independent equations of the set (5). Taking again the i-th and j-th equations as the independent ones one obtains:

$$f_i'' + \frac{f_i'}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{jK}\cos\theta_{iK}}{m_K L_K} + \frac{f_j'}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{Ki}\cos\theta_{iK}}{m_K L_K} = 0$$

$$f_j'' + \frac{f_i'}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{jK}\cos\theta_{Kj}}{m_K L_K} + \frac{f_j'}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{Ki}\cos_{jK}}{m_K L_K} = 0 \tag{13}$$

The first of these gives

$$f_j' = - \frac{f_i'' + \frac{f_i'}{\sin_{ji}} \sum_{K=1}^{n} \sin\theta_{jK}\cos\theta_{iK}/m_K L_K}{\frac{1}{\sin\theta_{ji}} \sum_{K=1}^{n} \sin\theta_{Ki}\cos\theta_{Ki}/m_K L_K} \tag{14}$$

Substitution into the second of eqs. (13) gives, after some algebraic manipulation and with the aid of relations (11),

$$f_i''' + P_1 f_i'' + P_2 f_i' = 0 \tag{15}$$

where the constants P_1 and P_2, independent of the choice of reference bars, are:

$$P_1 = \sum_{K=1}^{n} \frac{1}{m_K L_K} \tag{15a}$$

$$P_2 = \frac{1}{2} \sum_{K=1}^{n} \sum_{p=1}^{n} \frac{\sin^2\theta_{Kp}}{m_K L_K m_p L_p} \tag{15b}$$

The initial conditions for eq. (15) are provided by known initial velocity and displacement of the joint; they are

$$f_i'(0) = \frac{V}{a}\cos\theta_{Vi}$$

$$f_i'(0) = \frac{V}{a}\cos\theta_{Vj} \tag{15c}$$

$$f(0) = 0$$

The final result is

$$f_i = \left(\frac{C_1}{r_1}\right)\left[e^{r_1(at - x_i)} - 1\right] + \left(\frac{C_2}{r_2}\right)\left[e^{r_2(at - x_i)} - 1\right] \tag{16}$$

where C_1 and C_2 are to be determined from the first two of (15c), and where

$$r_{1,2} = \frac{1}{2}\left[-P + \sqrt{P_1^2 - 4P_2}\ \right] \tag{16a}$$

Type (b). For bars other than the p-th one, eqs. (8) show that the compatibility condition (12) may be simplified as in eq. (12a), namely

$$f'_{pi} \sin\theta_{jk} + f'_{pj} \sin\theta_{ki} + f'_{pk} \sin\theta_{ij} = 0 \quad ; \quad p \neq i,j,k \tag{17}$$

If p = i, say, then use of eqs. (8b) gives

$$(f'_{pp} - f'_p) \sin\theta_{jk} + f'_{pj} \sin\theta_{kp} + f'_{pk} \sin\theta_{pj} = 0 \tag{17a}$$

With use of (17), (17a) and (8c), the basic equations (6) can be reduced to:

$$\frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{jk}\cos\theta_{ik}}{m_K L_K} + \frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{ki}\cos\theta_{ik}}{m_K L_K} = \frac{2 f'_p}{m_p L_p} \cos_{pi}$$

$$\frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{jk}\cos\theta_{jk}}{m_K L_K} + \frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{ki}\cos\theta_{jk}}{m_K L_K} = -\frac{2 f'_p}{m_p L_p} \cos\theta_{pj} \tag{17b}$$

with i,j $\neq$ p. For the case i = p, the quantity f'_{pi} appearing in these equations must of course be replaced by $(f'_{pp} - f'_p)$. The relation between f'_{pi} and f'_p is the desired distribution factor; it can be obtained by algebraic solution of (17) and is listed in Table I.

<u>Type (c)</u>. The basic equations for this type of joint are again (5), where the velocities and stresses are respectively given by eqs. (8b) and (8c). The solution could be obtained by a treatment similar to that for joints of types (a) and (b); such an analytical solution, however, would become very complicated except in the simplest examples, and it was therefore considered more practical, in the work which follows, to employ a finite-difference technique for this type of joint. Both methods are however presented at this point.

The analytical solution gives, similarly to eq. (15):

$$f'''_{pi} + P_1 f''_{pi} + P_2 f'_{pi} = F_{pi} \qquad i \neq p \tag{18}$$

where

$$F_{pi} = -\frac{2}{m_p L_p}\left[f''_p \cos\theta_{ip} + \sum_{K=1}^{n} \frac{\sin\theta_{kp}\sin\theta_{ki}}{m_K L_K} f'_p\right] \tag{18a}$$

If i = p, again replace f'_{pi} by $(f'_{pp} - f'_p)$.

Let t_0 be the time at which the incoming wave arrives at this joint.

Then the solution of eq. (23), satisfying the conditions that the velocity of the striking mass is continuous at t = t_0, is

$$f' = a^2 e^{-r_1 at} \int_{t_0}^{t} \left\{ e^{(r_1 - r_2)at} \int_{t_0}^{t} e^{r_2 at} F_{pi}\, dt \right\} dt \tag{18b}$$

where r_1 and r_2 are given in eqs. (16a).

The procedure for the finite-difference solution is as follows. The basic equations (see eqs. (13) and (17c)) are:

$$f''_{pi} + \frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{jK}\cos\theta_{iK}}{m_K L_K} + \frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{Ki}\cos\theta_{iK}}{m_K L_K} = -\frac{2f'_p}{m_p L_p}\cos\theta_{pi} \tag{19}$$

$$f''_{pi} + \frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{jK}\cos\theta_{jK}}{m_K L_K} + \frac{f'_{pi}}{\sin\theta_{ji}} \sum_{K=1}^{n} \frac{\sin\theta_{Ki}\cos\theta_{jK}}{m_K L_K} = -\frac{2f'_p}{m_p L_p}\cos\theta_{pj}$$

for ij $\neq$ p. Let now

$$f''_{pi} \quad \left[f'_{pi}(\tau + \Delta\tau) - f'_{pi}(\tau)\right] \Big/ \Delta\tau \tag{19a}$$

and similarly for f''_{pj}. Employing the average values of the functions f'_{pi} and f'_{pj} with each interval $\Delta\tau$, one obtains the final finite-difference equations:

$$f'_{pi}(\tau+\Delta\tau)\left\{1+\frac{\Delta\tau}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{jK}\cos\theta_{iK}}{m_K L_K}\right\} + f'_{pj}(\tau+\Delta\tau)\left\{\frac{\Delta\tau}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{Ki}\cos\theta_{Ki}}{m_K L_K}\right\}$$

$$= -\Delta\tau\left\{\frac{f'_p(\tau+\Delta\tau)+f'_p(\tau)}{m_p L_p}\cos\theta_{ip} - \left[\frac{1}{\Delta\tau} - \frac{1}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{jK}\cos\theta_{iK}}{m_K L_K}\right] f'_{pi}(\tau)\right.$$

$$\left. + \frac{f'_{pj}(\tau)}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{Ki}\cos\theta_{Ki}}{m_K L_K}\right\} \tag{19b}$$

$$f'_{pi}(\tau+\Delta\tau)\left\{\frac{\Delta\tau}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{jK}\cos\theta_{jK}}{m_K L_K}\right\} + f'_{pj}(\tau+\Delta\tau)\left\{1+\frac{\Delta\tau}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{Ki}\cos\theta_{Ki}}{m_K L_K}\right\}$$

$$= -\Delta\tau\left\{\frac{f'_p(\tau+\Delta\tau)+f'_p(\tau)}{m_p L_p}\cos\theta_{jp} + \frac{f'_{pi}(\tau)}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{jK}\cos\theta_{jK}}{m_K L_K}\right.$$

$$\left. - \left[\frac{1}{\Delta\tau} - \frac{1}{2\sin\theta_{ji}}\sum_{K=1}^{n}\frac{\sin\theta_{Ki}\cos\theta_{jK}}{m_K L_K}\right] f'_{pj}(\tau)\right\}$$

Note that, similarly to eq. (17c), the quantity f'_{pi} appearing in eqs. (19) must be replaced by $(f'_{pp} - f'_p)$ when i = p. From this point on the solution is purely numerical and it is therefore postponed to the discussion of the illustrative example (Step 6.)

Type (d). A joint on a roller support has only one degree of freedom; this type of constraint is obtained by letting the direction k in eq. (12) be that of the reaction; then $u_k = 0$ and

$$\frac{u_i}{\sin\theta_{iR}} = \frac{u_j}{\sin\theta_{jR}} = \cdots\cdots = \frac{u_n}{\sin\theta_{nR}} \tag{20}$$

Similarly to eq. (12a) one may write

$$\frac{f'_{pi}}{\sin\theta_{iR}} = \frac{f'_{pi}}{\sin\theta_{jR}} = \frac{f'_{pp} - f'_p}{\sin\theta_{pR}} = \cdots\cdots = \frac{f_{pi}}{\sin\theta_{nR}} \tag{20a}$$

Selecting again two independent equations from the governing set

$$\sum_{j=1}^{n} \sigma_j A_j \cos\theta_{ij} + R\cos\theta_{iR} \tag{20b}$$

one may solve for the unknown reaction R and stress in the independent bar by purely algebraic manipulation. The result for the latter is given in Table I, and for the former is

$$R = \frac{-2Ef'_p \sum_{i=1}^{n} \sin\theta_{iR} \sin\theta_{pi} A_i A_p}{\sum_{i=1}^{n} (\sin^2\theta_{iR}) A_i} \tag{20c}$$

The angle between the i-th bar and the reaction is denoted by θ_{iR}.

Type (e). This type of joint does not displace; therefore, all values of u_i and σ_i vanish with the exception of σ_p which can be obtained from (8b) as

$$f'_{pp} > f'_p \tag{21}$$

The corresponding distribution factors are listed in Table I. The reaction is equal and opposite to $\sigma_p A_p$.

3. Criterion for the End of Impact

Consider a rigid mass M moving in space with a certain velocity; and assume that its uniform motion is disturbed by a collision with an elastic object, in this case the joint of a truss. As a consequence the mass velocity will change both in magnitude and direction, and will be in fact identical with that of the joint. Eventually, however, the mass will leave the joint either by rebounding or by slipping off. The time at which this happens is the time of end of impact; the determination of it is the object of the discussion below.

Immediately after the start of impact, the mass and joint move together in some curvilinear path; that is at any one time the mass is restrained from escaping along the tangent to that path with its instantaneous velocity (as it tends to do because of its inertia) by the interference provided by the truss. Should that interference suddenly end, the mass will of course leave and impact will cease.

The meaning of the term "interference," as employed above, may be clarified as follows. Consider a joint of type (a) or (c), and assume that the striking mass hits that joint within the angle defined by the i-th and the (i + 1)th bars, that is $0 \leq \theta_{iV} \leq \theta_{j,i+1}$. Let the joint and mass be moving, at a certain instant $t = t_0$, with a velocity V, and investigate their subsequent motions on the hypothesis that impact ends at that instant. The mass will then travel in a straight line a distance (Vdt) during a short interval dt. The joint will be one of type (b) during that interval and will therefore travel along a curve which, in general, will not be a straight line. Thus, the final positions of the joint and of the mass usually do not coincide. Consider now bars i and (i + 1); their position will have changed slightly during the time dt, and it will happen that, in certain cases, the above displacement of the mass places it outside the angle defined by these bars. As the mass was originally taken to be within this angle, the situation just described is physically impossible, and one may then conclude that interference exists. In such a case therefore impact will continue. On the other hand, should no interference arise between the position of the mass and that of the truss, the calculations mentioned above would yield the actual motion of the system, and impact would have ended. The above discussion is illustrated in Figs. 3 and 4 for the example of the truss of Fig. 1.

The following criterion for the time of end of impact may now be written; a necessary and sufficient condition for impact to continue is the persistence of interference as defined above; or, in other words, impact will end the first time that interference is not present.

Practical use of this criterion may be quite laborious for some instances; the calculations may be shortened by observing that interference will always exist when the component of the force F (on the mass) between mass and joint parallel to the direction of the mass velocity V has the same sense as V, i.e. when

$$F \cdot V \leqq 0 \tag{22}$$

where $\cdot$ indicates the scalar product. It is thus sufficient to limit the investigation of interference tot the times at which relation (22) fails, i.e. when

$$F \cdot V = 0 \tag{22a}$$

It is of interest to note some special problems in which the limiting case of (22a) holds. The most important of these is the example of longitudinal impact on the free end of a single bar. In this case $F \cdot V < 0$ until the time of maximum deflection, when $V = 0$ and therefore $F \cdot V = 0$. However, inspection shows that interference is always present and hence impact persists. As the mass rebounds, relation (22) holds until $F = 0$; at this time again $F \cdot V = 0$, and impact will end. A similar situation will arise in all cases in which the direction of the mass velocity (and force) is unchanged during impact, as is for instance the case whenever a symmetrical truss is subjected to impact in its plane of symmetry.

4. Numerical Solution and Illustrative Example

The behavior of truss of Fig. 1 is studied below. This truss allows no simplifications such as might be due to conditions of symmetry or a too restricted number of bars and of joints, and furthermore contains joints of all five types discussed earlier. The loading is applied by collision with a solid object; a solution corresponding to a prescribed force would be considerably simpler and is discussed more briefly later.

The proposed numerical solution consists of the steps outlined below.

Step 1. Make a scale drawing of the truss in question and list all the mechanical and geometrical parameters required in the solution. For the present example this is done in Fig. 1.

Step 2. List the distribution factors D_{ip} for all bars at all joints, as in Table II. They are obtained from Table I for joints of types (b), (d) and (e). The distribution factors listed in Table II for joint B (loaded joint) will be needed for the calculation of the time of end of impact, and if the example is to be continued beyond that time.

Step 3. Prepare (2n) tables, where n is the number of bars meeting at the struck joint; here n = 3. Of these tables one-half will be used for outgoing (travelling away from the loaded joint) waves, and will be labeled Tables O1, O2, O3, ..., On, respectively, for waves travelling along 1, 2, 3,..., n; the other half will be used for incoming (travelling toward the loaded joint), waves, and will be labeled Table I1, I2, I3,..., In, respectively, for waves travelling along bars 1, 2, 3..., n. The six tables required in the present problem are included in this paper. The first column of these tables gives the time in non-dimensional form as $\tau = (at/L)$, where L is a characteristic length and was here chosen as the length of the shortest bar, namely L = 1. It is convenient to list the quantity τ in equal intervals of magnitude $\Delta\tau$, chosen in such a way that the quantity (L_i/L) is, for all bars, and integral multiple of $\Delta\tau$; L_i denotes the length of the i-th bar. In the present example $\Delta\tau = 0.2$.

Step 4. This step corresponds to phase (a) of section 1. As soon as impact starts, the loaded joint (B) is of type (a); find then from eq. (16) the functions f' for any two (say the i-th and j-th) of the bars meeting at that joint; the values of f'_k for the k-th bar are then calculated from relation (12a). The final numerical results are:

$$\begin{aligned} \left(\frac{a}{V}\right) f'_1(\tau) &= 0.04356\, e^{-r_1\tau} + 0.67810\, e^{-r_2\tau} \\ \left(\frac{a}{V}\right) f'_2(\tau) &= -0.04246\, e^{-r_1\tau} + 0.69775\, e^{-r_2\tau} \\ \left(\frac{a}{V}\right) f'_3(\tau) &= -0.05936\, e^{-r_1\tau} + 0.00133\, e^{-r_2\tau} \end{aligned} \tag{23}$$

where

$$r_1 = -1.02501\ ; \quad r_2 = -0.47499 \tag{24}$$

The values of these f' functions are entered (for the appropriate values of τ) in the second solumn of Tables O1, O2, O3. These quantities represent all the waves present at the loaded joint until the first reflected wave arrives.

The stresses σ_i and velocities ($\partial u_i/\partial t$), during this period, can then be easily found from eqs. (3) and (4).

Step 5. In this step, corresponding to phase (b) of section 1, Tables I1, I2, ..., Ii,..., In are filled in according to eqs. (9) and (10) and the discussion following them, for all times τ for which this can be done without consideration of any joints of type (c). Consider as an example wave f'_{342} of Table I2. This wave arrives at joint B at $\tau = (L_3 + L_4 + L_2)/L = 3.6$; hence, all entries for it are zero for $0 < \tau < 3.6$. Now the first reflected wave to reach joint B is that travelling along the shortest bar and is in this example f'_{11}; it arrives at B when $\tau = (2L_1/L) = 2$; at that time it is divided (joint of type (c), see Step 6.) among bars 1, 2 and 3; in particular this last fraction of f'_{11} will be felt as part of f'_{342} as an incoming wave at joint B when $\tau = 3.6 + (2L_1/L) = 5.6$. Hence, entries for f'_{342} in Table I2 can be filled in during this step for the range $3.6 < \tau < 5.6$. The value $\tau = 5.6$ is indicated in that table by a heavy horizontal line, and similarly for all other waves. Note that all entries are zero for $0 < \tau < (2L_i/L)$ in Table Ii, where L_i represents the length of bar i.

The sum of all incoming waves is written in the last column on the right in these tables; it has been denoted by the symbol f'_{Ii} for Table Ii.

Step 6. This step corresponds to phase (c) of section 1. The loaded joint is considered as one of type (c), or in other words the distribution of an incoming wave at a loaded joint is taken up. Such a distribution can no longer be effected, as in step 5 for joints of type (b), simply by multiplication by a distribution factor; use of eqs. (19b) is therefore suggested. In the present case these equations reduce to (with i = 2, j = 3 and $\Delta\tau = 0.2$):

$$10.47577 f'_{PB2}(\tau+.2) + .39282 f'_{PB3}(\tau+.2) = -\left[f'_p(\tau+.2) + f'_p(\tau)\right]\cos\theta_{2P} -$$

$$-9.52423 f'_{PB2}(\tau) + 0.39282 f'_{PB3}(\tau) \qquad (25)$$

$$.00106 f'_{PB2}(\tau+.2) + 11.02425 f'_{PB3}(\tau+.2) = \left[f'_p(\tau+.2) + f'_p(\tau)\right]\cos\theta \quad +$$

$$.00106 f'_{PB2}(\tau) - 8.97575 f'_3(\tau)$$

where p = 1, 2 or 3 depending on the bar travelled by the incoming wave. For that joint eq. (12a) is:

$$.69986 f'_{PB1} - .67837 f'_{PB2} + .99875 f'_{PB3} = 0 \qquad (25a)$$

The notation f'_{pBi} denotes the influence on bar i of the total wave (f'_{Ip} of Table I_p) entering joint B through bar p. Note that, according to eqs. (17) f'_{pBp} must be replaced by ($f'_{pBp} - f'_{Ip}$). In the details of the numerical solution, the quantity ($-f'_{Ip}$) is entered first in tables Op. Then the quantities f'_{pBi} are calculated by simultaneous solution of eqs. (25) and subsequent use of (25a); the results are entered in Tables Oi. Note that the initial conditions for eqs. (25) are obtained from the fact that there cannot be, at a loaded joint, any sudden change in velocity; therefore, all functions

$$f'_{pBi} \quad \text{for } i \neq p \qquad [\text{and } (f'_{pBp} - f'_{Ip}) \text{ if } i = p] \tag{26}$$

must vanish at the time of arrival of the incoming wave.

Step 7. The sum of all the columns in Tables Oi, for the rows which have been completed, is now listed in the last column on the right in each of these tables. It is denoted as f'_{Oi}, and represents the total outgoing wave in bar i at the loaded joint. The stress σ_i and the velocity ($\partial u_i/\partial t$) in bar i at that joint are then (see eqs. (8)):

$$(\sigma_i \frac{a}{EV}) = -f'_{Ii} - f'_{oi} \tag{26a}$$

$$(\frac{1}{a})(\frac{\partial u_i}{\partial t}) = -f'_{Ii} - f'_{oi} \tag{26b}$$

These quantities may be calculated for all times by means of the previously described steps. Their values in the present problem are listed in Table III, and the stresses in bars 1, 2 and 3 are plotted in Fig. 5.

The resultant velocity V of the loaded joint, the resultant force F acting on the striking mass, and the angles θ_{Vi} and θ_{Fi} between bar i and these quantities are also given in Table III. They were calculated from the following formulas:

$$V^2 \sin^2\theta_{ij} = \left(\frac{\partial u_i}{\partial t} - \frac{\partial u_j}{\partial t}\cos\theta_{ij}\right)^2 + \left(\frac{\partial u_j}{\partial t}\sin\theta_{ij}\right)$$

$$\cos\theta_{vi} = \frac{1}{V}\frac{\partial u_i}{\partial t} \tag{26c}$$

$$F^2 = \left(\sum_{j=1}^{n} \sigma_j A_j \cos\theta_{ij}\right)^2 + \left(\sum_{j=1}^{n} \sigma_j A_j \sin\theta_{ij}\right)^2$$

$$\tan\theta_{Fi} = \frac{\sum_{j=1}^{n} \sigma_j A_j \sin\theta_{ij}}{\sum_{j=1}^{n} \sigma_j A_j \cos\theta_{ij}} \tag{26d}$$

The displacement δ of the loaded joint at any time is

$$\vec{\delta} = \sum \vec{V} \Delta t \tag{27}$$

where the terms under the summation must be added vectorially, a process most conveniently carried out graphically. In the present numerical examples the average value of the velocity within each interval Δ t was employed. A plot of the displacement of the loaded joint is shown in Fig. 3.

Duration of Impact. It has been shown that impact can end only when relation (22a) holds; in the present example this is true for $0 < \tau < 2.7$ and

$4.7 < \tau$. These limiting values are approximate; they correspond to the time at which

$$\tan \theta_{Vi} \tan \theta_{Fi} + 1 = 0 \tag{28}$$

and can be obtained by inspection of Table III. The final determination of the time of end of impact requires use of the definition of interference previously defined and is illustrated in Figs. (4a) and (4b), respectively for presence and absence of interference. The velocities of the mass and joint are required for drawing these figures and are given by eq. (27a); the quantity $(\partial u_i / \partial t)$ is given by eq. (26b) in the case of the mass, and by the relation

$$\left(\frac{1}{a}\right)\left(\frac{\partial u_i}{\partial t}\right)_{joint} = -f'_{Ii} + \sum_{j=1}^{n} D_{ij} f'_{Ij} \tag{29}$$

in the case of the joint (considered temporarily as one of type (b)). The approximate time of end of impact for the illustrative example is $\tau = 4.9$.

Interior Joints and Bars. The calculation of the stresses, velocities and displacements in the interior of the truss can now be discussed. This portion contains no loaded joints, and therefore the use of the previous finite-difference solution is no longer necessary; in fact, repetition of step 5 for the joint currently considered is all that is required.

Consider as an example the bars meeting at joint C. The wave $f'_{02}(\tau_o)$ appearing in Table 02 is also the total wave entering joint C from bar 2 at $\tau = \tau_o + 1.2$, and is therefore known. Similarly the total outgoing wave in that bar at joint C at $\tau = \tau_o - 1.2$ appears in Table I2 as $f'_{I2}(\tau_o)$. These two waves are entered in Table IV; as in eqs. (26) the stress and the velocity can be evaluated at this end of bar 2; they are also listed in that table.

For the other bars at joint C, the incoming and outgoing waves must be found from the procedure described in step 5. This step makes use of the distribution factors; as these have already been listed in Table II, no difficulties arise because of supported joints such as A and E (Fig. 1). The deflections of an interior joint and the stresses and velocities arising in all bars meeting are calculated in the same manner as for the loaded joint; some of these are listed in Table V. Plots of these and additional quantities (deflections of joint C and reactions, for example) have been presented elsewhere but appeared too lengthy to be repeated here. The solution may thus be considered complete.

5. Additional Remarks

Some additional information is presented here concerning checks on the numerical calculations, possible simplifications and the adaptation to other loading conditions.

1. In the calculations carried out at any one joint it is necessary to choose two bars to be considered as independent, and then to calculate the waves in the remaining bars from eq. (12a). The choice of independent bars is arbitrary; therefore, a check is obtained by deriving the same results with different independent coordinates. Another check is provided by the fact that the vector sum of the forces in all the bars meeting at an interior joint must be zero.

2. A certain number of joints must be travelled to reach any one bar from the loaded joint; as this number increases, the stresses decrease. (see for example Table V). This is so because each wave is distributed at each joint according to distribution factors which are smaller than unity; in other words, the truss configuration at large distances from the loaded joint influences little the stresses near the load. In the truss of Fig. 1, for example, impact has already ended when waves which passed through joint E return to the loaded joint.

3. It follows that the maximum stresses will usually occur in one of the bars meeting at the loaded joint. An upper limit for these stresses can be obtained by considering all joints adjacent to the loaded one as fixed hinges. Since joint A is of such a type in the truss of Fig. 1, this procedure gives in this particular case the actual maximum stress.

4. If the loading consists of a prescribed force varying with time, joints of type (a) and (c) are eliminated and therefore no finite-difference solution is necessary. The modifications necessary to reduce the loaded joint to one of type (b) have been mentioned in the discussion of that type of joint.

5. If impact occurs at more than one joint, either simultaneously or not, the stresses and velocities can be obtained by superposing the values obtained by considering each impact separately. Care must be exercised, however, that it is necessary to consider together the effects of all loadings when the time of end of impact for any one of the striking masses is calculated.

6. In the calculations previously described only the stresses and velocities at the ends of each bar were determined. It is easy to find the values of these quantities say at a distance x_i from one of the ends (say M) of bar i, with the aid of the expressions

$$\sigma(\tau, x_i) = \sigma_{\text{outgoing}}\left(\tau = \frac{x_i}{L}\right) + \sigma_{\text{incoming}}\left(\tau + \frac{x_i}{L}\right)$$

$$\frac{\partial}{\partial t}\left[u_i(\tau, x_i)\right] = \frac{\partial}{\partial t}\left[u_i\left(\tau - \frac{x_i}{L}\right)\right]_{\text{outg.}} - \frac{\partial}{\partial t}\left[u_i\left(\tau + \frac{x_i}{L}\right)\right]_{\text{inc.}}$$

where the subscripts indicate the total outgoing or incoming wave at $x_i = 0$, i.e. joint M.

7. The striking mass was assumed perfectly rigid in the present example so as not to introduce any complications extraneous to the truss proper. Should the mass be elastic, some modifications in the calculations pertaining to joints of type (a) would be required, perhaps employing the Hertz theory of impact for the determination of the impulsive force.

6. Equations for Space Trusses

The procedure described above for the analysis of trusses under impact is quite general and can therefore be applied directly to three-dimensional trusses; some of the basic equations, however, must be modified to take into account the additional degree of freedom. The principal equations are listed below and are derived with the aid of some well known formulas of vector analysis.[6] Note that all equations of section 1 are still valid. Eq. (12) for the three-dimensional case becomes

6. Wills, A. P.: "Vector Analysis, with an Introduction to Tensor Analysis," Sections 94-95, Prentice-Hall, Inc., New York, 1931.

$$u_K = c_K^1 u_1 + c_K^2 u_2 + c_K^3 u_3 \tag{30}$$

where bars 1, 2 and 3 are not coplanar, and where

$$c_K^i = g^{i1} \cos\theta_{1K} + g^{i2} \cos\theta_{2K} + g^{i3} \cos\theta_{3K} \tag{30a}$$

$$\begin{aligned}
&g^{11} = \frac{1}{g} \sin^2\theta_{23} \; ; \; g^{22} = \frac{1}{g} \sin^2\theta_{13} \; ; \; g^{33} = \frac{1}{g} \sin^2\theta_{12} \\
&g^{12} = g^{21} = \frac{1}{g} \left(\cos\theta_{23} \cos\theta_{13} = \cos\theta_{12}\right) \\
&g^{13} = g^{31} = \frac{1}{g} \left(\cos\theta_{12} \cos\theta_{23} - \cos\theta_{13}\right) \\
&g^{23} = g^{32} = \frac{1}{g} \left(\cos\theta_{13} \cos\theta_{12} - \cos\theta_{23}\right)
\end{aligned} \tag{30b}$$

$$g = 1 - \cos^2\theta_{23} - \cos^2\theta_{12} - \cos^2\theta_{13} + 2\cos\theta_{12} \cos\theta_{13} \cos\theta_{23} \tag{30c}$$

Eqs. (13), in the three-dimensional case, are replaced by the following equations:

$$f_i'' + f_i' \sum \frac{c_p^i}{m_p L_p} \cos\theta_{ip} + f_j' \sum \frac{c_p^j}{m_p L_p} \cos\theta_{ip} + f_K' \sum \frac{c_p^K}{m_p L_p} \cos\theta_{ip} = 0 \tag{31}$$

and two other similar equations for bars j and k, where, in the summations, p = 1, 2, 3,....n. Eq. (15) becomes in this case

$$f'''' + P_1 f_i''' + P_2 f_i'' + P_3 f_i' = 0 \tag{32}$$

where the quantities P_1 and P_2 are identical with those given by eqs. (15a) and (15b) respectively, and

$$P_3 = \frac{1}{3!} \sum_{s=1}^{n} \sum_{q=1}^{n} \sum_{r=1}^{n} \frac{g_{sqr}}{m_s L_s \, m_q L_q \, m_r L_r} \tag{32a}$$

and where

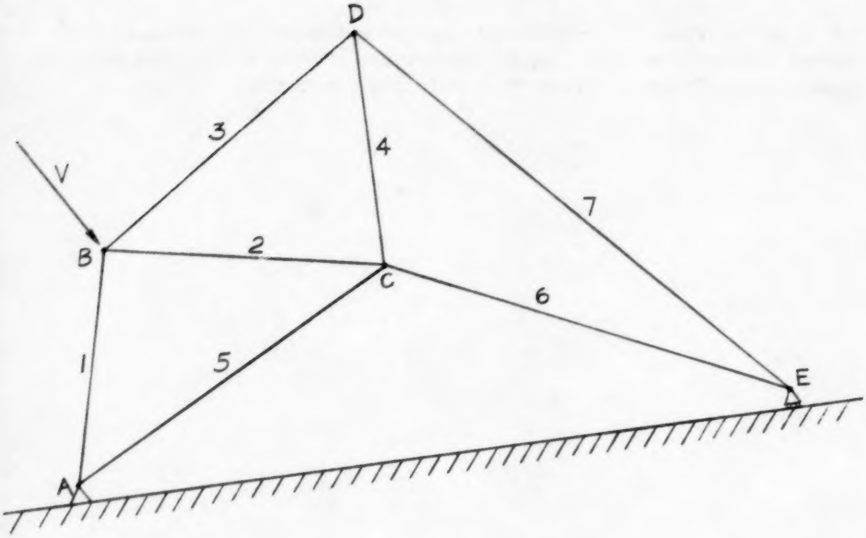
$$g_{sqr} = 1 - \cos^2\theta_{sq} - \cos^2\theta_{qr} - \cos^2\theta_{rs} + 2\cos\theta_{sq} \cos\theta_{qr} \cos\theta_{rs} \tag{32b}$$

Note that $g = g_{123}$, and that $g_{sqr} = g_{srq} = g_{rqs}$ and so forth.
With the above equations, joints of type (a) can be treated. For joints of type (b) the necessary distribution factor D_{ip} of eq. (9) becomes

$$D_{ip} = \frac{1}{P_3} \sum_{q=1}^{n} \sum_{r=1}^{n} \frac{\left(g_{iqr} \, g_{pqr}\right)^{\frac{1}{2}}}{m_i L_i \, m_q L_q \, m_r L_r} \qquad i \neq p \tag{33}$$

$$D_{ii} = 1 - \frac{1}{P_3} \sum_{q=1}^{n} \sum_{r=1}^{n} \frac{q_{iqr}}{m_i L_i \, m_q L_q \, m_r L_r} \tag{33a}$$

For joints of types (c) the finite-differences technique should again be employed; the pertinent equations (corresponding to (19b) are cumbersome but otherwise easily derived and are therefore not presented.



$$A_1 = A_2 = A_3 = A_4 = A_5 = A_6 = A_7 = 1$$

$$L_1 = \frac{L_2}{1.2} = \frac{L_3}{1.4} = L_4 = \frac{L_5}{1.6} = \frac{L_6}{1.8} = \frac{L_7}{2.4} = 1$$

$$m = 2 \quad m = 5/3 \quad m_3 = 10/7 \quad m_4 = 2 \quad m_5 = 5/8 \quad m_6 = 10/9 \quad m_7 = 5/6$$

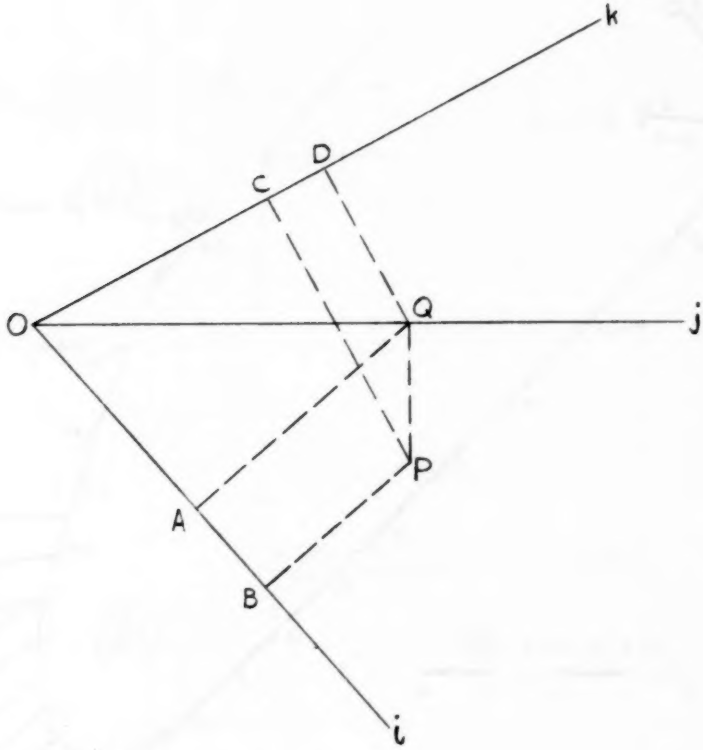
$$m_1 L_1 = m_2 L_2 = m_3 L_3 = m_4 L_4 = m_5 L_5 = m_6 L_6 = m_7 L_7 = 2$$

$$\theta_{12} = 92^\circ 52' \qquad \theta_{23} = 44^\circ 25' \qquad \theta_{25} = 38^\circ 37\tfrac{1}{2}$$

$$\theta_{42} = 78^\circ 28' \qquad \theta_{64} = 114^\circ 58\tfrac{1}{2} \qquad \theta_{76} = 22^\circ 11\tfrac{1}{2}$$

$$\theta_{V1} = -43^\circ 48\tfrac{1}{2}$$

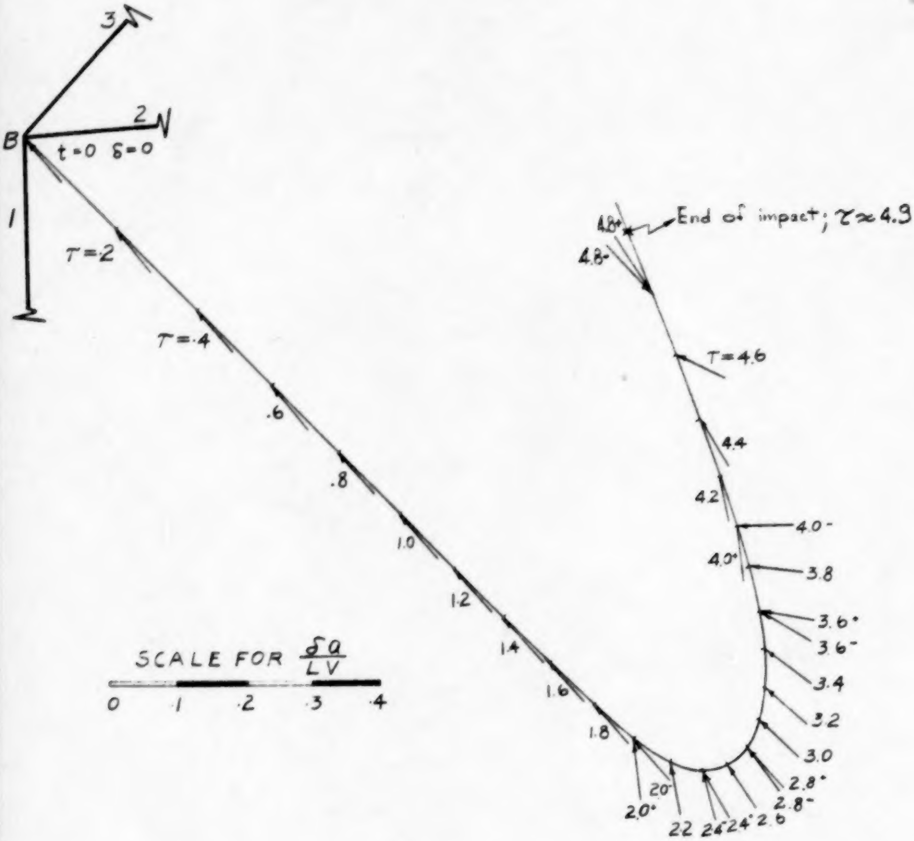
Fig. 1. Truss for Illustrative Example.



$$PQ \sin\theta_{ij} = AB = OB - OA = u_i - u_j \cos\theta_{ij}$$

$$PQ \sin\theta_{jk} = CD = OD - OC = u_j \cos\theta_{jk} - u_k$$

Fig. 2. Displacement of a Joint in a Plane Truss. Original Position at 0; Final Position at P.



(ARROW SHOWS DIRECTION OF FORCE, NOT TO SCALE)

Fig. 3. Displacements of Joint B of the Truss of Fig. 1.

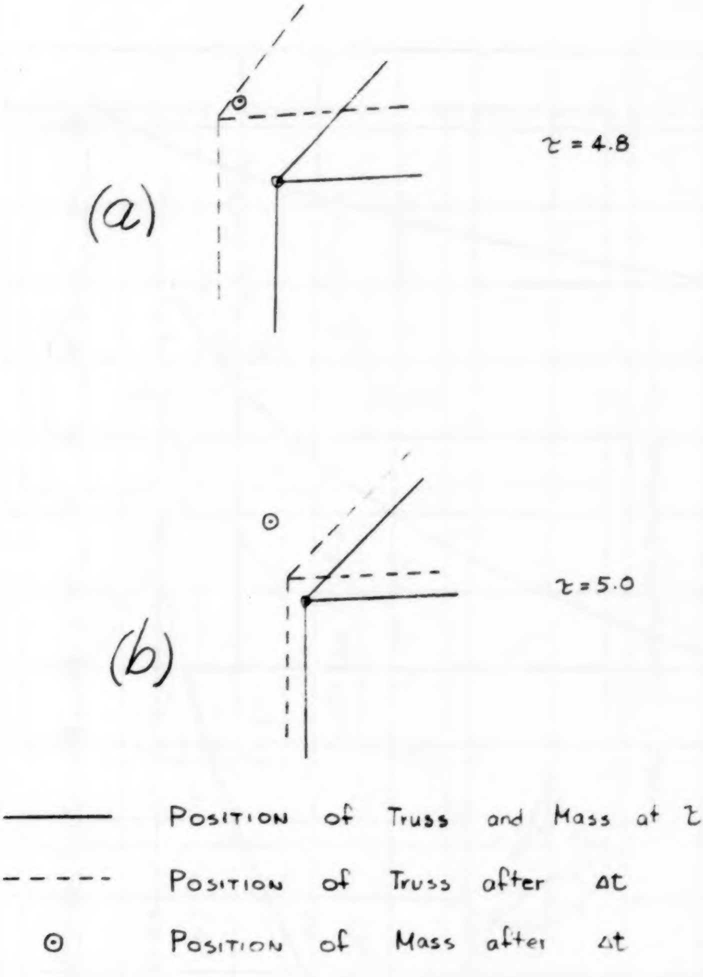


Fig. 4. Determination of Time of End of Impact.
a) impact continues
b) impact ends

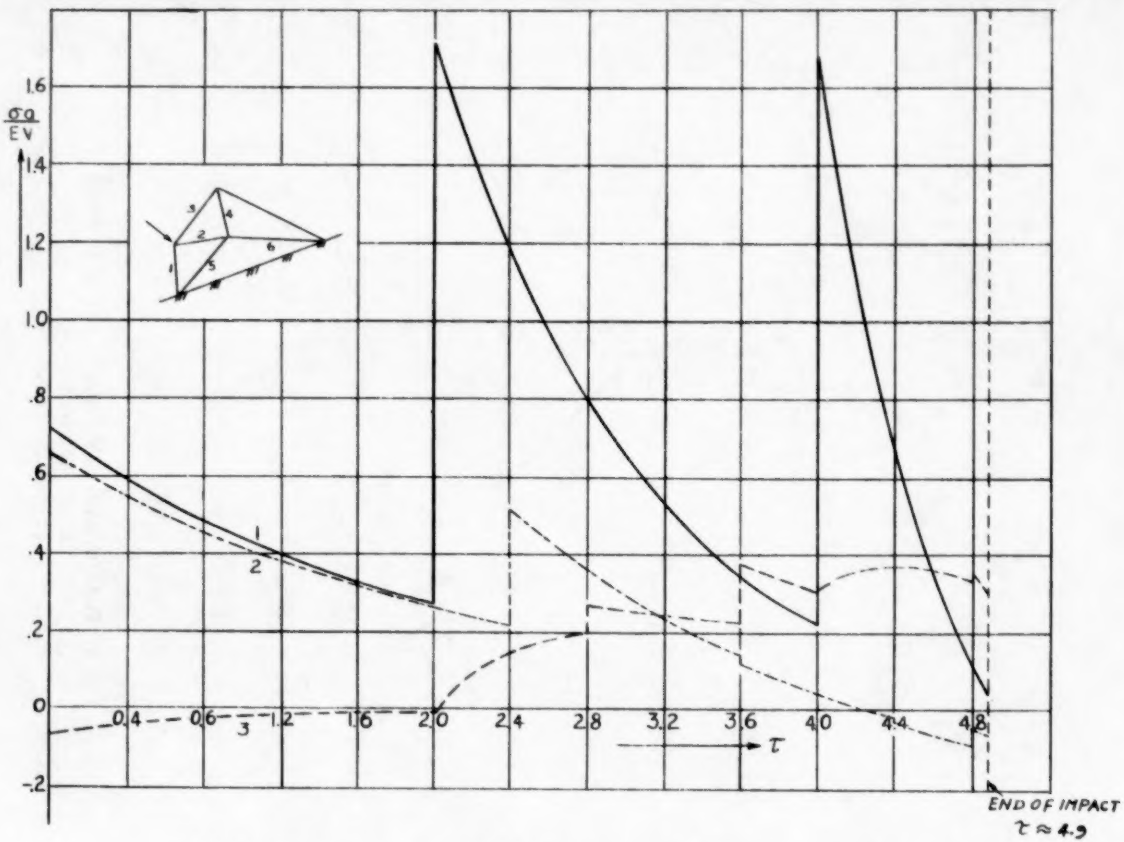


Fig. 5. Stresses at Joint B in Bars 1, 2, and 3 of the Truss of Fig. 1.

Joint	$D_{pi} \quad (i \neq p)$	D_{pp}
Type (b)	$-4 \dfrac{\sum_{k=1}^{n} A_p A_k \sin\theta_{ik} \sin\theta_{pk}}{\sum_{k=1}^{n} \sum_{q=1}^{n} A_k A_q \sin^2\theta_{kq}}$	$1-4 \dfrac{\sum_{k=1}^{n} A_p A_k \sin^2\theta_{pk}}{\sum_{k=1}^{n} \sum_{q=1}^{n} A_k A_q \sin^2\theta_{kq}}$
Type (d)	$-2 \dfrac{A_p \sin\theta_{pR} \sin\theta_{iR}}{\sum_{j=1}^{n} A_j \sin^2\theta_{jR}}$	$1-2 \dfrac{A_p \sin^2\theta_{pR}}{\sum_{j=1}^{n} A_j \sin^2\theta_{jR}}$
Type (e)	0	1

TABLE I. Equations for Distribution Factors.

	p \ i	1	2	3	4	5	6	7
JOINT A	1	1	-	-	-	0	-	-
	5	0	-	-	-	1	-	-
JOINT B	1	-.49699	-.48756	.71783	-	-	-	-
	2	-.48756	-.52741	-.69579	-	-	-	-
	3	.71783	-.69579	.02440	-	-	-	-
JOINT C	2	-	.22876	-	-.18956	-.58000	.75850	-
	4	-	-.18956	-	-.41442	.72887	.51094	-
	5	-	-.58000	-	.72887	.00864	.36368	-
	6	-	.75850	-	.51094	.36368	.17703	-
JOINT D	3	-	-	-.56753	-.62654	-	-	.53419
	4	-	-	-.62654	-.09230	-	-	-.77390
	7	-	-	.53419	-.77390	-	-	-.34017
JOINT E	6	-	-	-	-	-	-.27420	-.96167
	7	-	-	-	-	-	-.96167	.27420

TABLE II. Distribution Factors D_{pi} for the Truss of Fig. 1.

τ	$\frac{\sigma_1 a}{E V}$	$\frac{\sigma_2 a}{E V}$	$\frac{\sigma_3 a}{E V}$	$F_{//1} \frac{a}{EV}$	$F_{\perp 1} \frac{a}{EV}$	$\tan \theta_{F1}$	θ_{F1}
0	.72166	.65529	-.06069	.73348	.61330	.83615	219°54'
.2	.65213	.59992	-.04957	.65855	.56554	.85877	220°39'
.4	.58967	.54883	-.04049	.59197	.52068	.87957	221°20'
.6	.53350	.50177	-.03309	.53272	.47870	.89859	221°57'
.8	.48291	.45846	-.02705	.47986	.43954	.91597	222°29'
1.0	.43733	.41869	-.02212	.43264	.40316	.93186	222°59'
1.2	.39622	.38219	-.01810	.39041	.36943	.94627	223°25'
1.4	.35911	.34873	-.01482	.35256	.33824	.95938	223°49'
1.6	.32558	.31808	-.01214	.31859	.30945	.97130	224°10'
1.8	.29527	.29004	-.00995	.28808	.28293	.98211	224°29'
2.0 -	.26786	.26438	-.00816	.26063	.25851	.99188	224°46'
2.0 +	1.71118	.26438	-.00816	1.70395	.25851	.15171	188°38'
2.2	1.41969	.24407	.08486	1.34514	.30133	.22401	192°38'
2.4 -	1.17490	.21895	.15180	1.05242	.32165	.30563	197°0'
2.4 +	1.17490	.51873	.15180	1.03743	.62106	.59865	210°54'
2.6	.96991	.43907	.17981	.81584	.56050	.68702	214°30'
2.8 -	.79629	.36638	.19708	.63317	.49962	.78907	218°17'
2.8 +	.79629	.36638	.26956	.57992	.54878	.94631	223°25'
3.0	.65352	.29666	.25679	.45002	.47049	1.04548	226°16'
3.2	.53184	.23529	.24591	.33940	.40181	1.18388	229°49'
3.4	.42823	.18144	.23390	.24131	.33988	1.37432	233°58'
3.6 -	.34022	.13595	.22216	.17020	.28649	1.68324	239°17'
3.6 +	.34022	.12153	.37786	.05652	.37771	6.68271	261°30'
3.8	.27564	.07499	.33885	.02293	.30476	13.29097	265°42'
4.0 -	.21996	.03587	.30425	-.00537	.24222	-45.10600	271°16'
4.0 +	1.66328	.03587	.30425	1.43745	.24222	.16851	189°34'
4.2	1.10531	.00603	.35653	.84306	.24788	.29403	196°23'
4.4 -	.67823	-.02490	.37292	.40548	.22811	.56257	209°22'
4.4 +	.67823	-.02924	.39292	.40570	.22377	.55158	208°53'
4.6	.35580	-.05648	.36514	.09035	.19129	2.11722	244°43'
4.8 -	.11196	-.08360	.34131	-.13463	.14804	-1.09961	312°17'
4.8 +	.11196	-.01514	.36105	-.15255	.22980	-1.50639	303°35'
5.0	-.06376	-.06141	.21947	-.22194	.08755	-.39448	338°28'

TABLE III. Magnitudes and Directions of Stresses, Velocities

$\frac{1}{V}\frac{\partial u_1}{\partial t}$	$\frac{1}{V}\frac{\partial u_2}{\partial t}$	$\frac{1}{V}\frac{\partial u_3}{\partial t}$	$\frac{1}{V}\frac{\partial u_{21}}{\partial t}$	$\frac{1}{V}\frac{\partial u}{\partial t}$	$\tan\theta_{v1}$	θ_{v1}	$\tan\theta_{F1}\tan\theta_{v1}$
.72166	.65529	-.06069	.69225	1.00000	.95925	43°48'	.80208
.65213	.59992	-.04957	.63332	.90905	.97116	44°10'	.83400
.58967	.54883	-.04049	.57904	.82644	.98197	44°29'	.86371
.53350	.50177	-.03309	.52911	.75139	.99177	44°46'	.89119
.48291	.45846	-.02705	.48321	.68315	1.00062	45°1'	.91653
.43733	.41869	-.02212	.44111	.62116	1.00864	45°15'	.93991
.39622	.38219	-.01810	.40251	.56481	1.01588	45°27'	.96130
.35911	.34873	-.01482	.36715	.51357	1.02239	45°38'	.98086
.32558	.31808	-.01214	.33478	.46699	1.02826	45°48'	.99875
.29527	.29004	-.00995	.30519	.42465	1.03360	45°57'	1.01511
.26786	.26438	-.00816	.27812	.38613	1.03830	46°5'	1.02987
.26786	.26438	-.00816	.27812	.38613	1.03830	46°5	.15752
.11543	.24407	.08486	.25016	.27551	2.16720	65°14'	.48547
-.00444	.21895	.15180	.21900	.21905	-49.32432	91°10'	-15.07499
-.00444	.21895	.15180	.21900	.21905	-49.32432	91°10'	-29.52800
-.09709	.16461	.17981	.15995	.18711	-1.64744	121°15'	-1.13182
-.16953	.11530	.19708	.10696	.20045	-.63092	147°45'	-.49784
-.16953	.11530	.19708	.10696	.20045	-.63092	147°45'	-.59705
-.22114	.06712	.20053	.05613	.22815	-.25382	165°46'	-.26536
-.26060	.02555	.19995	.01253	.26090	-.04808	177°15'	-.05692
-.28999	-.01010	.19634	-.02463	.29104	.08493	184°51'	.11675
-.31094	-.03889	.19146	-.05451	.31568	.17531	189°57'	.29509
-.31094	-.03889	.19146	-.05451	.31568	.17531	189°57'	1.17155
-.31490	-.07277	.17121	-.08863	.32713	.28145	195°43'	3.74074
-.31576	-.10003	.15331	-.11597	.33638	.36727	200°10'	-16.56608
-.31576	-.10003	.15331	-.11597	.33638	.36727	200°10'	.06189
-.42981	-.11879	.22049	-.14046	.45218	.32680	198°6'	.09609
-.49223	-.13942	.25020	-.16424	.50891	.33367	198°27'	.18771
-.49223	-.13942	.25020	-.16424	.50891	.33367	198°27'	.18404
-.51702	-.15890	.25436	-.18499	.54912	.35780	199°41'	.75754
-.51480	-.17594	.24123	-.20194	.55300	.39277	201°25'	-.43185
-.51480	-.17594	.24123	-.20194	.55300	.39277	201°25'	-.59166
-.49614	-.19265	.21681	-.21773	.43885	.43885	203°42'	-.17312

and Force at Joint B in Bars 1, 2, and 3 of the Truss of Fig. 1.

τ	$f_{inc.}$	$f_{outg.}$	$\frac{a}{EV}\sigma_2$	$\frac{1}{V}\frac{\partial u_2}{\partial t}$
1.2	.65529	.14989	.80518	.50540
1.4	.59992	.13723	.73715	.46269
1.6	.54883	.12554	.67437	.42329
1.8	.50177	.11477	.61654	.38700
2.0	.45846	.10487	.56333	.35359
2.2	.41869	.09577	.51446	.32292
2.4	.38219	.08742	.46961	.29477
2.4	.38219	.08021	.46240	.30198
2.6	.34873	.07388	.42261	.27485
2.8	.31808	.06795	.38603	.25013
3.0	.29004	.06241	.35245	.22763
3.2	.26438	.05726	.32164	.20712
3.2	.26438	.05509	.31947	.20929
3.4	.24407	.05121	.29528	.19286
3.6	.21895	.04617	.26512	.17278
3.6	.36884	.08040	.44924	.28844
3.8	.30184	.06562	.36746	.23622
4.0	.24084	.05213	.29297	.18871
4.2	.18189	.03904	.22093	.14285
4.4	.13042	.02759	.15801	.10283
4.4	.13042	.24774	.37816	-.11732
4.6	.08567	.23010	.31577	-.14443
4.8	.04853	.21251	.26104	-.16398
4.8	.04132	.10747	.14879	-.06615
5.0	.00111	.09463	.09574	-.09352

TABLE IV. Stresses and Velocities in Bar 2 at Joint C of the Truss of Fig. 1.

τ	$\frac{\sigma_2 a}{EV}$	$\frac{\sigma_5 a}{EV}$	$\frac{\sigma_6 a}{EV}$	$\frac{\sigma_4 a}{EV}$	$F_{\parallel 2}$	$F_{\perp 2}$
1.2	.80518	-.38006	.49704	-.12422	0	0
1.4	.73715	-.34795	.45504	-.11372		
1.6	.67437	-.31832	.41629	-.10404		
1.8	.61654	-.29102	.38059	-.09512		
2.0	.56333	-.26590	.34774	-.08691		
2.2	.51446	-.24284	.31758	-.07937		
2.4	.46961	-.22167	.28989	-.07245		
2.4	.46240	-.19395	.30932	-.05019		
2.6	.42261	-.17962	.28038	-.04792		
2.8	.38603	-.16600	.25422	-.04544		
3.0	.35245	-.15311	.23059	-.04284		
3.2	.32164	-.14099	.20919	-.04019		
3.2	.31947	-.13263	.21505	-.03347		
3.4	.29528	-.12381	.19759	-.03200		
3.6	.26512	-.11172	.17677	-.02924		
3.6	.44924	-.19866	.29047	-.05766		
3.8	.36746	-.16190	.23818	-.04664		
4.0	.29297	-.12830	.19067	-.03649		
4.2	.22093	-.09562	.14489	-.02654		
4.4	.15801	-.06704	.10495	-.01781		
4.4	.37816	-.44932	-.03253	-.29397		
4.6	.31577	-.43408	-.08499	-.29672		
4.8	.26104	-.41377	-.12421	-.29307		
4.8	.14879	-.45916	-.29010	-.36133		
5.0	.09574	-.41738	-.30642	-.33862		
5.2	.05061	-.37726	-.31579	-.31527		
5.2	.05461	-.39268	-.32660	-.32764		
5.4	.01856	-.35082	-.32446	-.30048		
5.6	-.01913	-.30880	-.32417	-.27381		
5.6	.00134	-.26657	-.27770	-.23572		
5.8	-.03736	-.23500	-.28535	-.21741	0	0

TABLE V. Magnitudes and Directions of Stresses, Velocities

$\frac{1}{V}\frac{\partial u_2}{\partial t}$	$\frac{1}{V}\frac{\partial u_5}{\partial t}$	$\frac{1}{V}\frac{\partial u_6}{\partial t}$	$\frac{1}{V}\frac{\partial u_4}{\partial t}$	$\frac{1}{V}\frac{\partial u_{22}}{\partial t}$	$\frac{1}{V}\frac{\partial u}{\partial t}$	$\tan\theta_{2u}$	θ_{2u}
.50540	-.38006	.49704	-.12422	-.02368	.50597	-.04685	-2°41′
.46269	-.34795	.45504	-.11372	-.02167	.46320	-.04683	
.42329	-.31832	.41629	-.10404	-.01983	.42375	-.04685	
.38700	-.29102	.38059	-.09512	-.01814	.38743	-.04687	
.35359	-.26590	.34774	-.08691	-.01657	.35398	-.04686	
.32292	-.24284	.31758	-.07937	-.01512	.32327	-.04682	
.29477	-.22167	.28989	-.07245	-.01381	.29509	-.04685	-2°41′
.30198	-.19395	.30932	-.12623	-.06724	.30938	-.22266	-12°33′
.27485	-.17962	.28038	-.11004	-.05624	.28054	-.20466	-11°34′
.25013	-.16600	.25422	-.09618	-.04712	.25453	-.18838	-10°40′
.22763	-.15311	.23059	-.08430	-.03961	.23105	-.17401	-9°52′
.20712	-.14099	.20919	-.07409	-.03336	.20979	-.16107	-9°9′
.20929	-.13263	.21505	-.09031	-.04947	.21506	-.23637	-13°18′
.19286	-.12381	.19759	-.08072	-.04303	.19760	-.22312	-12°35′
.17278	-.11172	.17677	-.07112	-.03727	.17676	-.21571	-12°10′
.28844	-.19866	.29047	-.09954	-.04275	.29159	-.14821	-8°26′
.23622	-.16190	.23818	-.08278	-.03628	.23899	-.15359	-8°44′
.18871	-.12830	.19067	-.06775	-.03065	.19118	-.16242	-9°14′
.14285	-.09562	.14489	-.05366	-.02560	.14513	-.17921	-10°10′
.10283	-.06704	.10495	-.04141	-.02130	.10501	-.20714	-11°42′
-.11732	.31080	-.03253	-.32047	-.35107	.37015	2.99241	251°31′
-.14443	.26182	-.08499	-.20496	-.23867	.27897	1.65250	238°49′
-.16398	.22287	-.12421	-.11593	-.15181	.22346	.92578	222°48′
-.06615	.17748	-.01752	-.18419	-.20153	.21211	3.04656	251°50′
-.09352	.16466	-.05688	-.12502	-.14674	.17401	1.56908	237°29′
-.11477	.15454	-.08749	-.07887	-.10393	.15483	.90555	222°10′
-.11877	.13912	-.09830	-.04896	-.07422	.14005	.62491	211°58′
-.13132	.13486	-.11574	-.02436	-.05168	.14113	.39362	201°29′
-.14513	.13454	-.13347	-.00373	-.03382	.14908	.23294	193°7′
-.17000	.12133	-.14934	.02912	.01839	.17099	-.10818	175°50′
-.17802	.12424	-.16211	.04019	.02377	.17960	-.13352	172°24′

and Force at Joint C in Bars 2, 4, 5 and 6 of the Truss of Fig. 1.

τ	f_{I1}
0	0
⋮	⋮
2.0-	0
2.0+	.72166
2.2	.65213
2.4	.58967
2.6	.53350
2.8	.48291
3.0	.43733
3.2	.39622
3.4	.35911
3.6	.32558
3.8	.29527
4.0-	.26786
4.0+	.98952
4.2	.76756
4.4	.58523
4.6	.43641
4.8	.31338
5.0	.21619

TABLE 11. Incoming Waves at Joint B in Bar 1.

τ	f_{33}	f_{243}	f_{3443}	f_{I3}
0				
⋮				
2.2				
2.4				
2.6				
2.8	.03444			.03444
3.0	.02813			.02813
3.2	.02298			.02298
3.4	.01878			.01878
3.6	.01535			.01535
3.6	.01535	.07785		.09320
3.8	.01255	.07127		.08382
4.0	.01027	.06520		.07547
4.2	.00841	.05961		.06802
4.4	.00689	.05447		.06136
4.6	.00565	.04974		.05539
4.8-	.00463	.04541		.05004
4.8+	.00463	.04541	.00987	.05991
5.0	-.04816	.04143	.00806	.00133

TABLE 13. Incoming Waves at Joint B in Bar 3.

T	f_{22}	f_{342}	f_{2442}	f_{34442}	f_{2552}	f_{2662}	f_{744442}	f_{3762}	f_{I2}
2.4	.14989								.14989
2.6	.13723								.13723
2.8	.12554								.12554
3.0	.11477								.11477
3.2	.10487								.10487
3.4	.09577								.09577
3.6	.08747								.08742
3.6	.08742	-.00721							.08021
3.8	.07977	-.00589							.07388
4.0	.07276	-.00481							.06795
4.2	.06634	-.00393							.06241
4.4	.06047	-.00321							.05726
4.4	.06047	-.00321	-.00217						.05509
4.6	.05583	-.00263	-.00199						.05121
4.8	.05008	-.00215	-.00182						.04611
4.8	.08437	-.00215	-.00182						.08040
5.0	.06904	-.00176	-.00166						.06562
5.2	.05509	-.00144	-.00152						.05213
5.4	.04161	-.00118	-.00139						.03904
5.6	.02983	-.00097	-.00127						.02759
5.6	.02983	-.00097	-.00127	-.00028	.22043				.24774
5.8	.01960	.01008	-.00116	-.00023	.20181				.23010
6.0	.01110	.01803	-.00106	-.00018	.18462				.21251
6.0	.00945	.01803	-.00106	-.00018	.18462	-.10339			.10747
6.2	.00025	.02135	-.00096	-.00015	.16879	-.09465			.09463
6.4	-.00734	.02340	-.00088	-.00012	.15422	-.08659			.08269
6.4	-.00734	.02749	-.00088	-.00012	.15422	-.08659	-.00009		.08669
6.6	-.01296	.02715	-.00081	-.00010	.14084	-.07916	-.00008		.07494
6.8	-.01879	.02647	-.00073	-.00008	.12856	-.07233	-.00007		.06303
6.8	-.01929	.02647	-.00122	-.00008	.12856	-.07233	-.00007	.02365	.08567
7.0	-.02463	.02554	-.00100	-.00007	.11730	-.06606	-.00007	.01931	.07033

TABLE 12. Incoming Waves at Joint B in Bar 2.

τ	f_1	f_{I1}	$f_{1B1} - f_{I1}$	f_{2B1}	f_{3B1}	f_{O3}
0	.72166					.72166
.2	.65213					.65213
.4	.58967					.58967
.6	.53350					.53350
.8	.48291					.48291
1.0	.43733					.43733
1.2	.39622					.39622
1.4	.35911					.35911
1.6	.32558					.32558
1.8	.29527					.29527
2.0 -	.26786					.26786
2.0 +	.26786	.72166	0			.98952
2.2	.24306	.65213	-.12763			.76756
2.4	.22059	.58967	-.22503	0		.58523
2.6	.20025	.53350	-.29799	.00065		.43641
2.8	.18181	.48291	-.35123	-.00011	0	.31338
3.0	.16510	.43733	-.38861	-.00181	.00418	.21619
3.2	.14995	.39622	-.41329	-.00407	.00681	.13562
3.4	.13620	.35911	-.42784	-.00668	.00833	.06912
3.6	.12374	.32558	-.43437	-.00936	.00905	.01464
3.8	.11242	.29527	-.43459	-.01190	.01917	-.01963
4.0 -	.10215	.26786	-.42990	-.01424	.02623	-.04790
4.0 +	.10215	.98952	-.42990	-.01424	.02623	.67376
4.2	.09282	.76756	-.53720	-.01636	.03093	.33775
4.4	.08435	.58523	-.59219	-.01819	.03380	.09300
4.6	.07666	.43641	-.60922	-.01977	.03531	-.08061
4.8	.06968	.31338	-.59919	-.02106	.03577	-.20142
5.0	.06334	.21619	-.57067	-.02196	.03315	-.27995

TABLE 01. Outgoing Waves at Joint B in Bar 1.

τ	f_2	f_{1B2}	f_{I2}	$f_{2B2} - f_{I2}$	f_{3B2}	f_{02}
0	.65529					.65529
.2	.59992					.59992
.4	.54883					.54883
.6	.50177					.50177
.8	.45846					.45846
1.0	.41869					.41869
1.2	.38219					.38219
1.4	.34873					.34873
1.6	.31808					.31808
1.8	.29004					.29004
2.0	.26438	0				.26438
2.2	.24095	.00312				.24407
2.4 -	.21952	-.00057				.21895
2.4 +	.21952	-.00057	.14989	0		.36884
2.6	.19998	-.00866	.13723	-.02671		.30184
2.8	.18213	-.01937	.12554	-.04746	0	.24084
3.0	.16585	-.03136	.11477	-.06332	-.00405	.18189
3.2	.15101	-.04369	.10487	-.07516	-.00661	.13042
3.4	.13747	-.05569	.09577	-.08379	-.00809	.08567
3.6 -	.12514	-.06690	.08742	-.08833	-.00880	.04853
3.6 +	.12514	-.06690	.08021	-.08833	-.00880	.04132
3.8	.11390	-.07703	.07388	-.09101	-.01863	.00111
4.0	.10366	-.08591	.06795	-.09230	-.02548	-.03208
4.2	.09433	-.09063	.06241	-.09246	-.03003	-.05638
4.4 -	.08584	-.10074	.05726	-.09170	-.03282	-.08216
4.4 +	.08584	-.10074	.05509	-.09170	-.03282	-.08433
4.6	.07810	-.11284	.05121	-.08989	-.03427	-.10769
4.8 -	.07106	-.12471	.04617	-.08757	-.03472	-.12977
4.8 +	.07106	-.12471	.08040	-.08757	-.03472	-.09554
5.0	.06465	-.13498	.06562	-.09016	-.03216	-.12703

TABLE 02. Outgoing Waves at Joint B in Bar 2.

τ	f_3	f_{1B3}	f_{2B3}	f_{I3}	$f_{3B3}-f_{I3}$	f_{03}
0	-.06069					-.06069
.2	-.04957					-.04957
.4	-.04049					-.04049
.6	-.03309					-.03309
.8	-.02705					-.02705
1.0	-.02212					-.02212
1.2	-.01810					-.01810
1.4	-.01482					-.01482
1.6	-.01214					-.01214
1.8	-.00995					-.00995
2.0	-.00816	0				-.00816
2.2	-.00669	.09155				.08486
2.4	-.00550	.15730	0			.15180
2.6	-.00452	.20293	-.01860			.17981
2.8	-.00372	.23296	-.03216			.19708
2.8	-.00372	.23296	-.03216	.03444	0	.23152
3.0	-.00306	.25101	-.04174	.02813	-.00568	.22866
3.2	-.00252	.25993	-.04820	.02298	-.00926	.22293
3.4	-.00208	.26198	-.05223	.01878	-.01133	.21512
3.6	-.00172	.25894	-.05344	.01535	-.01232	.20681
3.6	-.00172	.25894	-.05344	.09320	-.01232	.28466
3.8	-.00143	.25221	-.05348	.08382	-.02609	.25503
4.0	-.00118	.24289	-.05271	.07547	-.03569	.22878
4.2	-.00098	.31488	-.05134	.06802	-.04207	.28851
4.4	-.00082	.34654	-.04954	.06136	-.04598	.31156
4.6	-.00068	.35026	-.04720	.05539	-.04802	.30975
4.8-	-.00057	.33517	-.04472	.05004	-.04865	.29127
4.8+	-.00057	.33517	-.04472	.05991	-.04865	.30114
5.0	-.00050	.30821	-.04583	.00133	-.04507	.21814

TABLE 03. Outgoing Waves at Joint B in Bar 3.